Sec 2 – Expansion & Factorization of Algebraic Expression

	TOPIC
3.1	Expanding Products of Algebraic Expressions
3.2	Expanding & Factorizing using Special Products
4.	Factorization by Grouping
4.2	Factorization by box method (For Quadratic Eqn)
5.	Algebraic Manipulation
5.1	Addition and Subtraction of Algebraic Fractions
5.2	Multiplication and Division of Algebraic Fractions
5.3	Change the Subject of a Formula
5.4	Solving Equations involving Algebraic Fractions

Name:	Date:

Main Ideas	Notes/Examples
Distributive Law of Multiplication (Expansion)	(a) $a(b + c) = ab + ac$ (b) $(a + b)(c + d) = ac + ad + bc + bd$ (c) $(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$
Special Products ▼▼▼▼▼▼▼▼▼▼▼▼ **SP1 and SP2 are only meant for <u>Perfect Square Trinomials.</u>** *How to* Recognize a Perfect Square Trinomial? ▪ First term is a **perfect square** ▪ Last term is a **perfect square** ▪ Middle term = **±2ab**	🟣 **SP 1 :** $(a + b)^2 = a^2 + 2ab + b^2$ <u>**Step-by-Step Method for Expanding**</u> Example: $(2x+7y)^2$ **Step 1: Square the first term** $\qquad (2x)^2 = 4x^2$ **Step 2: Double the product of the two terms** $\qquad 2(2x)(7y) = 28xy$ ***Step 3: Square the last term*** $\qquad (7y)^2 = 49y^2$ **Step 4: Combine all terms** $\qquad (2x+7y)^2 = 4x^2 + 28xy + 49y^2$ <u>**Step-by-Step Method for Factorisation**</u> Example: $9x^2 + 6xy + y^2$ **Step 1: Square root the first term** = $\sqrt{9x^2} = \underset{a}{3x}$ **Step 2: Square root the last term** = $\sqrt{y^2} = \underset{b}{y}$ **Step 3: Check that the middle term matches SP2 pattern**. $\qquad 2(3x)(y) = 6xy$ ✓ **Ans:** $(\underset{a}{3x} + \underset{b}{y})^2$

SP 2 : $(a - b)^2 = a^2 - 2ab + b^2$

Step-by-Step Method for Expanding

Example: $(2x - 7y)^2$

Step 1: Square the first term
$$(2x)^2 = 4x^2$$

Step 2: Double the product of the two terms

(-2ab)
$$-2(2x)(7y) = -28xy$$

Step 3: Square the last term
$$(7y)^2 = 49y^2$$

Step 4: Combine all terms
$$(2x - 7y)^2 = 4x^2 - 28xy + 49y^2$$

Step-by-Step Method for Factorisation

Example: $25x^2 - 10xy + y^2$

Step 1: Square root the first term: $25a^2 = (5a)^2$

Step 2 : Square root the last term: $b^2 = (b)^2$

Step 3: Check that the middle term matches SP2 pattern
$$-2(5a)(b) = -10ab \checkmark$$

Ans: $(5a + b)^2$

● **SP 3 : $a^2 - b^2 = (a+b)(a-b)$**

Step-by-Step Method for Factorisation

Example 1 : $9x^2 - 16y^2$

Step 1: Make sure that for both terms, the coefficient and variable of x & y are squared (raised to the power of 2).

$$9x^2 = \underbrace{(3x)^2}_{a} \text{ and } 16y^2 = \underbrace{(4y)^2}_{b}$$

$$a^2 - b^2 = (a+b)(a-b) = (3x+4y)(3x-4y)$$

Example 2 : $8x^2 - 18y^2$

What happens when one of the term is not a perfect square ? Bring our the common factor first !

$$2(4x^2 - 9y^2)$$

Step 1: $4x^2 = \underbrace{(2x)^2}_{a} \text{ and } 9y^2 = \underbrace{(3y)^2}_{b}$

Ans: $2(2x+3y)(2x-3y)$

Example 3 : $1 - 16y^2$

Student must know that $1 = 1^2$

Step 1: $1 = \underbrace{1^2}_{a} \text{ and } 16y^2 = \underbrace{(4y)^2}_{b}$

Ans: $(1+4y)(1-4y)$

Practice on Applying **SP1 & SP2** (Basic -Expansion)	Expand each of the following expressions using Special Product Rules.	
	1. $(2x - 9y)^2$	**2.** $(15a + 4b)^2$
Recall: $\quad$ **SP 1 :** $(a + b)^2 = a^2 + 2ab + b^2$ $\quad$ **SP 2 :** $(a - b)^2 = a^2 - 2ab + b^2$	**3.** $\left(\dfrac{1}{7}x^2 - 9y\right)^2$	**4.** $\left(\dfrac{1}{5}a + \dfrac{2}{3}b^2\right)^2$
	Without Using a calculator, evaluate using Algebraic rules. (a) $99^2 + 198 + 1$ (b) $49^2 + 98 + 1$ (c) 295^2 (d) 305^2	

Practice on Applying **SP1 & SP2** (Basic - Factorisation) Recall: **SP 1 :** $(a+b)^2 = a^2 + 2ab + b^2$ **SP 2 :** $(a-b)^2 = a^2 - 2ab + b^2$	Factorise each of the following expressions using Special Product Rules.	
	1. $p^2 - 2pq + q^2$	**2.** $9x^2 - 6xy + y^2$
	3. $4p^2 + 12pq + 9q^2$	**4.** $9p^2q^2 - 30pq + 25$

Practice on Applying **SP3** (Basic - Factorisation)	Factorise each of the following expressions using Special Product Rules.	
Recall: $SP\ 3 : a^2 - b^2 = (a+b)(a-b)$	**1.** $(3p)^2 - q^2$	**2.** $9p^2 - 100$
	3. $49p^2 - 16$	**4.** $1 - q^2$
	5. $4x^2 - 25$	**6.** $200x^2 - 2$
	7 **2019 O level EM P1 [1]** $x^2 - y^4$	**8** **2023 O level EM P1 [1]** $(2x+1)^2 - (x+3)^2$

Without Using a calculator, evaluate using Algebraic rules.

(a) 301×299

(b) 992×1008

(c) $19^2 - 1^2$

(d) $121^2 - 21^2$

Application Questions

2025 Methodist Girls School S4 PRELIM P1 Q10a [2marks]
1. Factorise completely. (a) $5 - 5x^2$ *Answer*.................................... [2]
2. 2022 ACS (Barker Rd) S2 EOY P1 Q3b [3marks]
$4x^2 - 16y^2$.   *Answer* _____ [2]
3. 2022 JUYING S2 EOY P2 Q2 [3marks]
n is a positive integer. Show that $(2n + 1)^2 - (2n - 3)^2$ is a multiple of 8 for all integer values of n. [3]

4. 2023 ZHONGHUA S2 EOY P1 Q15b [5marks]

(i) Factorise completely $4x^2 - 12xy + 9y^2$

Answer _____ [1]

(ii) Hence, simplify $\dfrac{72y^2 - 8x^2}{36y^2 - 24xy + 4x^2}$.

Answer _____ [3]

Ans: (b) $\dfrac{2(3y+x)}{(3y-x)}$

5. 2020 ST JOSEPH INSTITUTION S2 EOY P1 Q4 [5marks]

(i) Factorise completely $4x^2 + 12xy + 9y^2$

Answer _____ [2]

(ii) Use your result in (a) to factorise $4x^2 + 12xy + 9y^2$-16 completely.

Answer _____ [2]

Ans: (a) $(2x+3y)^2$ (b) $(2x+3y-4)(2x+3y+4)$

Application Questions

2025 Methodist Girls School S4 PRELIM P1 Q10a [2marks]

1.

Factorise completely.

(a) $5 - 5x^2$

$$5(1 - x^2)$$
$$= 5(1^2 - x^2)$$
$$= 5(1-x)(1+x)$$

Answer ...... $5(1-x)(1+x)$ ...... [2]

2. 2022 ACS (Barker Rd) S2 EOY P1 Q3b [3marks]

$4x^2 - 16y^2 = (2x)^2 - (4y)^2$
$$= (2x+4y)(2x - 4y)$$

Answer $(2x+4y)(2x - 4y)$ [2]

3. 2022 JUYING S2 EOY P2 Q2 [3marks]

n is a positive integer. Show that $(2n + 1)^2 - (2n - 3)^2$ is a multiple of 8 for all integer values of n. [3]

$a^2 - b^2 = (a+b)(a-b)$
$$= (2n+1 + 2n-3)(2n+1 - (2n-3))$$
$$= (4n - 2)(1 - (-3))$$
$$= (4n-2)(1+3)$$
$$= (4n-2)(4)$$
$$= 16n - 8 = 8(2n-1)$$

Since 8(2n-1) is a factor of 8, the equation $(2n + 1)^2 - (2n - 3)^2$ is a multiple of 8 for all integer values of n.

4. 2023 ZHONGHUA S2 EOY P1 Q15b [5marks]

(i) Factorise completely $72y^2 - 8x^2$

$$8(9y^2 - x^2)$$
$$8((3y)^2 - x^2)$$
$$8(3y - x)(3y + x)$$

Answer ___$8(3y-x)(3y+x)$___ [1]

(ii) Hence, simplify $\dfrac{72y^2 - 8x^2}{36y^2 - 24xy + 4x^2}$ ← $\sqrt{36y^2} = 6y$, $\sqrt{4x^2} = 2x$, $-2(6y)(2x) = -24yx$.

↑ use Ans from (i)

$$\frac{8(3y-x)(3y+x)}{(6y-2x)^2} = \frac{8(3y-x)(3y+x)}{(6y-2x)(6y-2x)}$$

$$= \frac{8(3y-x)(3y+x)}{2(3y-x)\,2(3y-x)} = \frac{8(3y+x)}{4(3y-x)}$$

Factorize 2 out, so that we can do cancellation.

Answer ___$\dfrac{2(3y+x)}{(3y-x)}$___ [3]

Ans: (b) $\dfrac{2(3y+x)}{(3y-x)}$

5. 2020 ST JOSEPH INSTITUTION S2 EOY P1 Q4 [5marks]

(i) Factorise completely $4x^2 + 12xy + 9y^2$

$\sqrt{4x^2} = 2x$ $\sqrt{9y^2} = 3y$

checker: $(2)(2x)(3y) = 12xy$

$\therefore (2x + 3y)^2$

Answer ___$(2x + 3y)^2$___ [2]

(ii) Use your result in (a) to factorise $4x^2 + 12xy + 9y^2 - 16$ completely.

$$(2x+3y)^2 - 16 \quad \leftarrow \text{** student must familiarize themselves w number that are perfect square}$$
$$\Rightarrow (2x+3y)^2 - 4^2 \quad \leftarrow a^2 - b^2$$
$$\Rightarrow (2x+3y+4)(2x+3y-4)$$

Answer ___$(2x+3y+4)(2x+3y-4)$___ [2]

Ans: (a) $(2x+3y)^2$ (b) $(2x+3y-4)(2x+3y+4)$